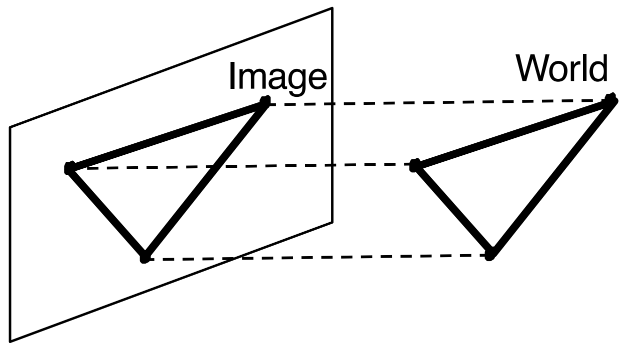


Folien zur Vorlesung am 25.03.2025

3D Computer Vision

PROJEKTION

Projection


MAKE POVERTY HISTORY
JULIAN BEEVER
CoolOpticalIllusions.com

Projection

Projection

Image credit: Prof. Dr. Stephan Neser

Projection

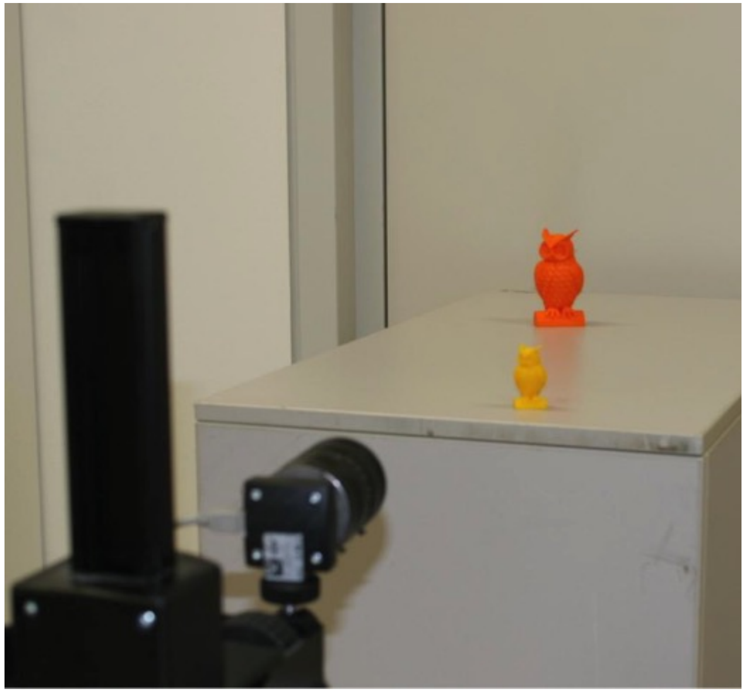
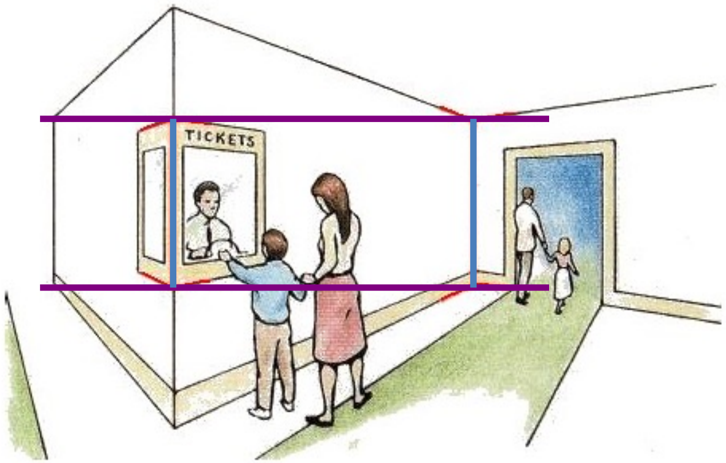


Image credit: Prof. Dr. Stephan Neser

Müller-Lyer Illusion



https://en.wikipedia.org/wiki/Müller-Lyer_illusion

Geometric Model: A Pinhole Camera

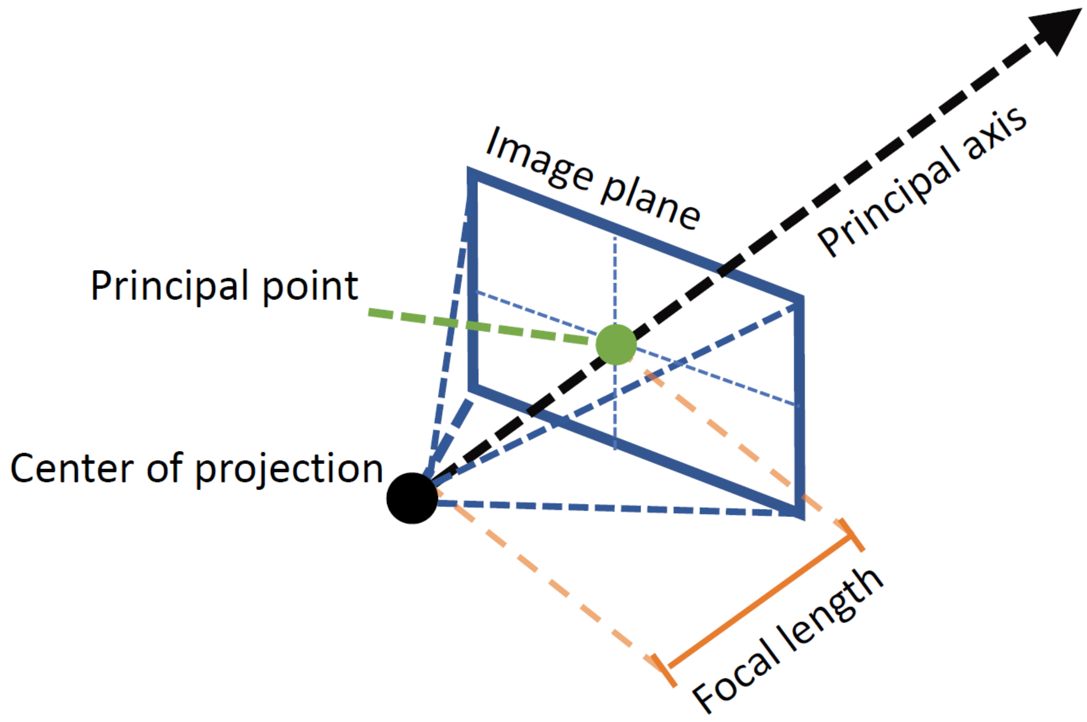
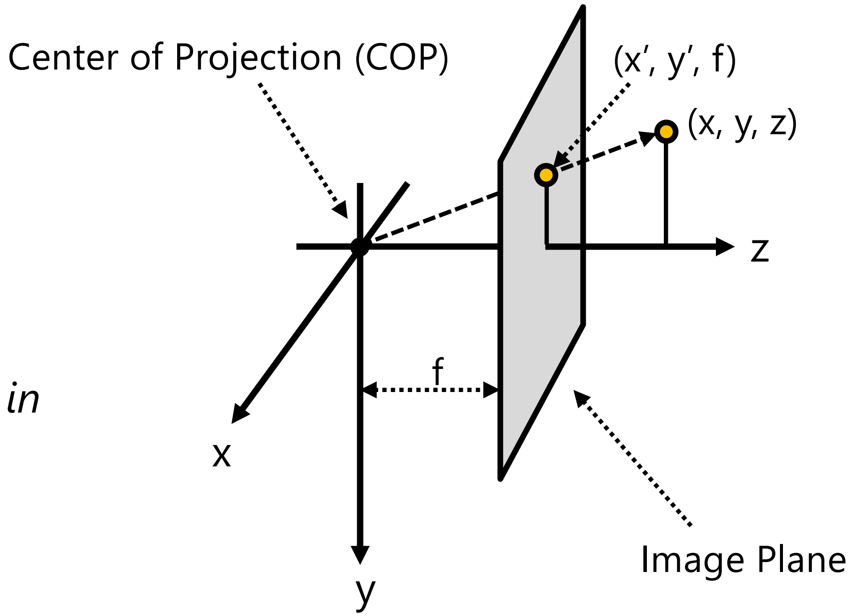


Figure credit: Peter Hedman

Modeling projection

- The coordinate system
  - We use the pinhole model as an approximation
  - Put the optical center (aka Center of Projection, or COP) at the origin
  - Put the Image Plane (aka Projection Plane) *in front* of the COP (Why)?
  - The camera looks down the *positive* z-axis, and the y-axis points down
    - we like this if we want right-handed-coordinates
    - other versions are possible (e.g., OpenGL)



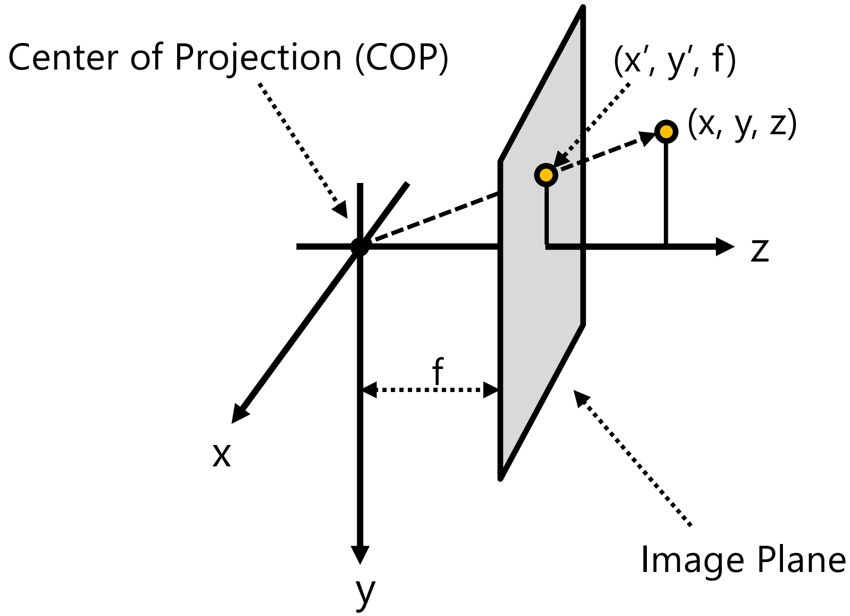
Modeling projection

- Projection equations
  - Compute intersection with Image Plane of ray from (x,y,z) to COP
  - Derived using similar triangles

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, f)$$

  - We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$



Modeling projection

- Is this a linear transformation?
  - **no** - division by z is nonlinear

Homogeneous coordinates to the rescue

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Position is lost due to projection

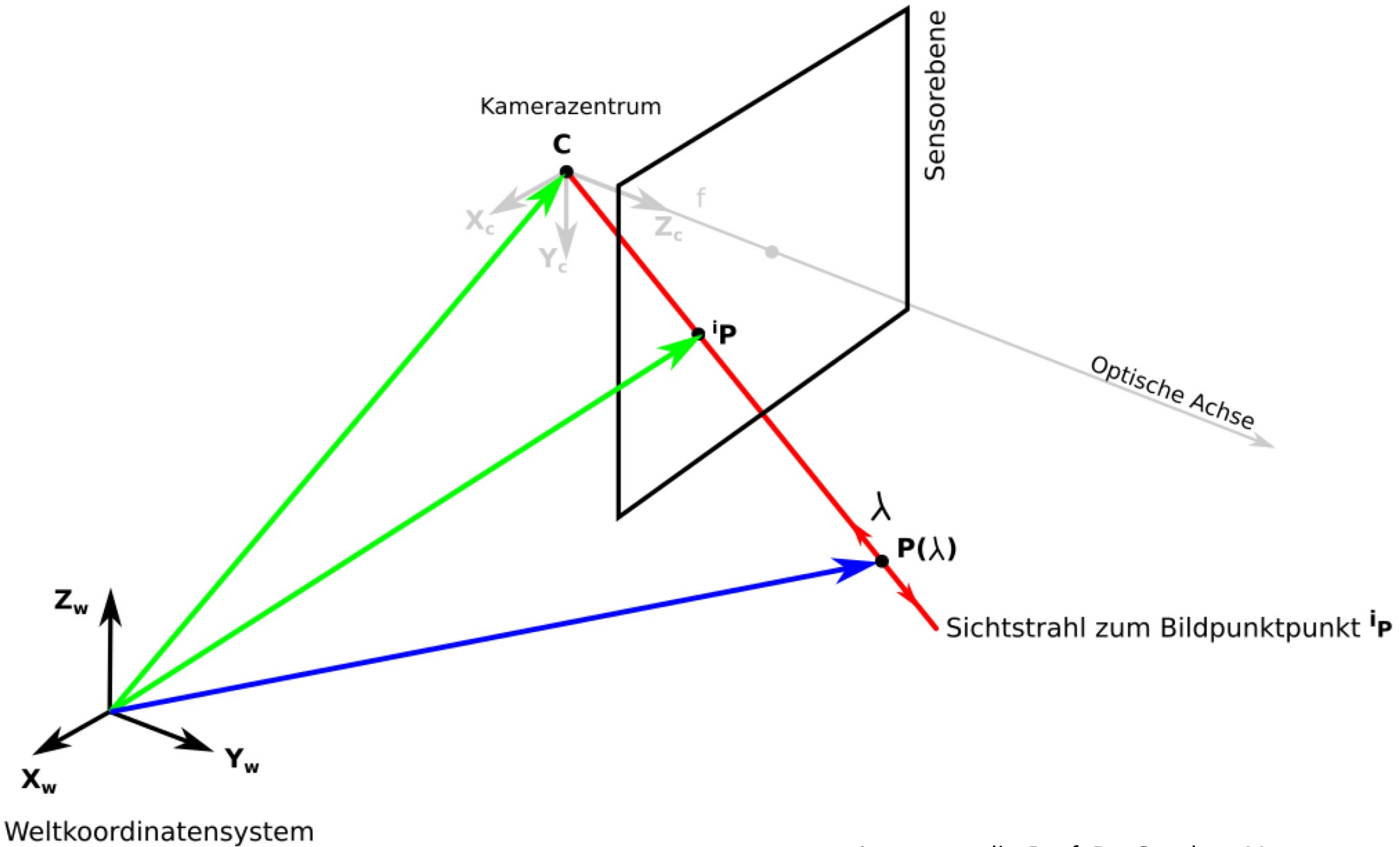


Image credit: Prof. Dr. Stephan Neser

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- (Can also represent as a 4x4 matrix – OpenGL does something like this)

Perspective Projection

How does scaling the projection matrix change the transformation?

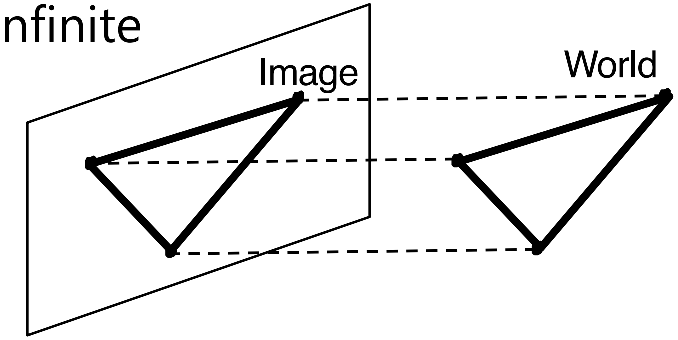
$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1/f & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix} = \begin{bmatrix}x\\ y\\ z/f\end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Scale by *f*: $$\begin{bmatrix}f & 0 & 0 & 0\\ 0 & f & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix} = \begin{bmatrix}fx\\ fy\\ z\end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Scaling a projection matrix produces an equivalent projection matrix!

Orthographic projection

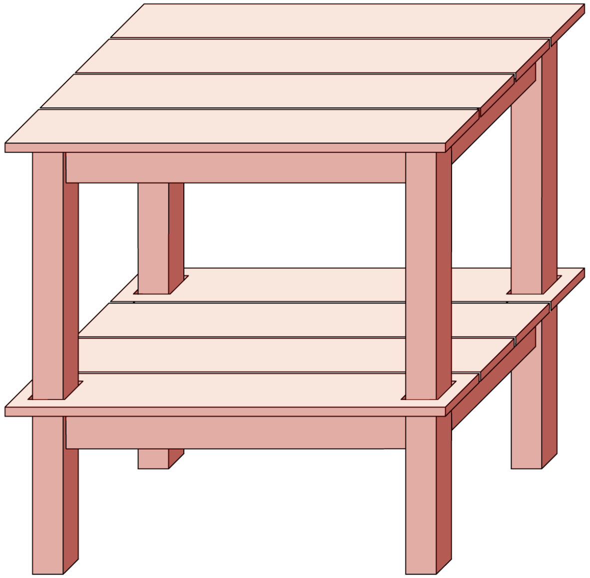
- Special case of perspective projection
  - Distance from the COP to the image plane is infinite



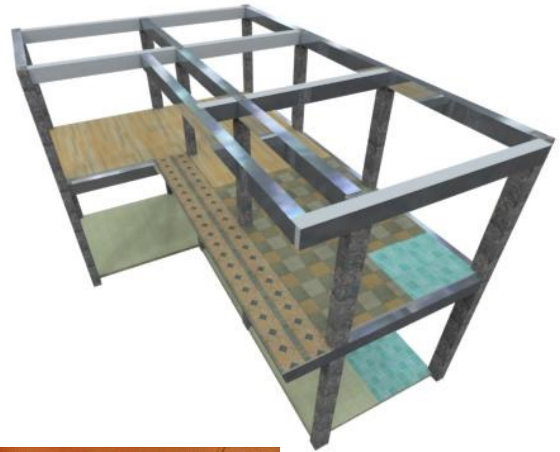
  - Good approximation for telephoto optics
  - Also called "parallel projection": (x, y, z) → (x, y)
  - What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic projection

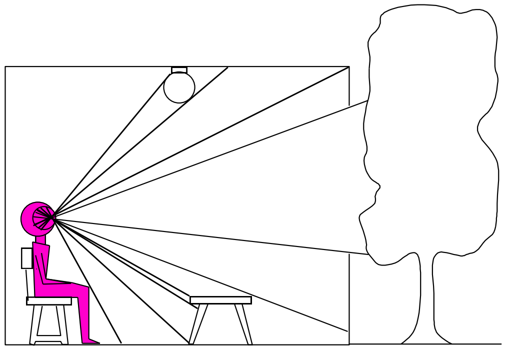


Perspective projection



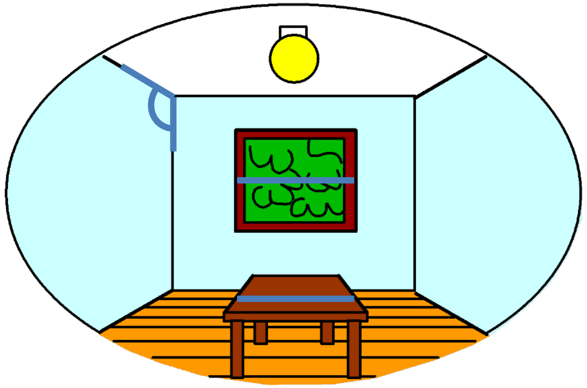
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points → points
- Lines → lines (collinearity is preserved)
  - But line through COP projects to a point
- Planes → planes (or half-planes)
  - But plane through COP projects to line

Projection properties

- Parallel lines converge at a vanishing point
  - Each direction in space has its own vanishing point
  - But parallel lines parallel to the image plane remain parallel