$$\mathbf{K} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

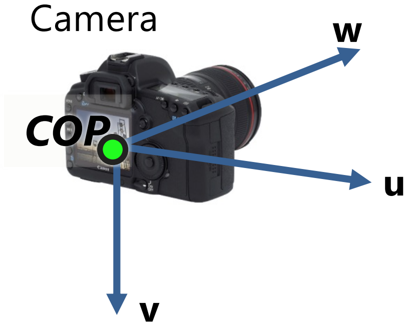
Folien zur Vorlesung am 01.04.2025

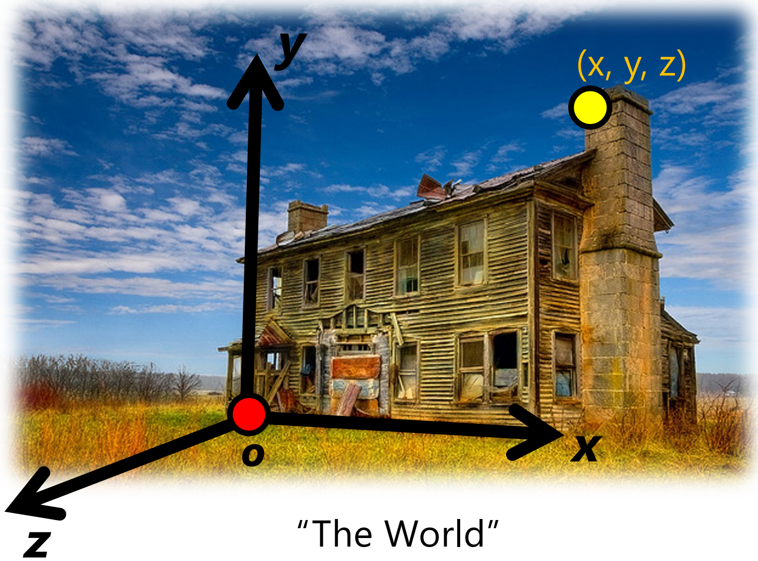
3D Computer Vision

KAMERAPARAMETER

Camera parameters

- How can we model the geometry of a camera?





"The World"

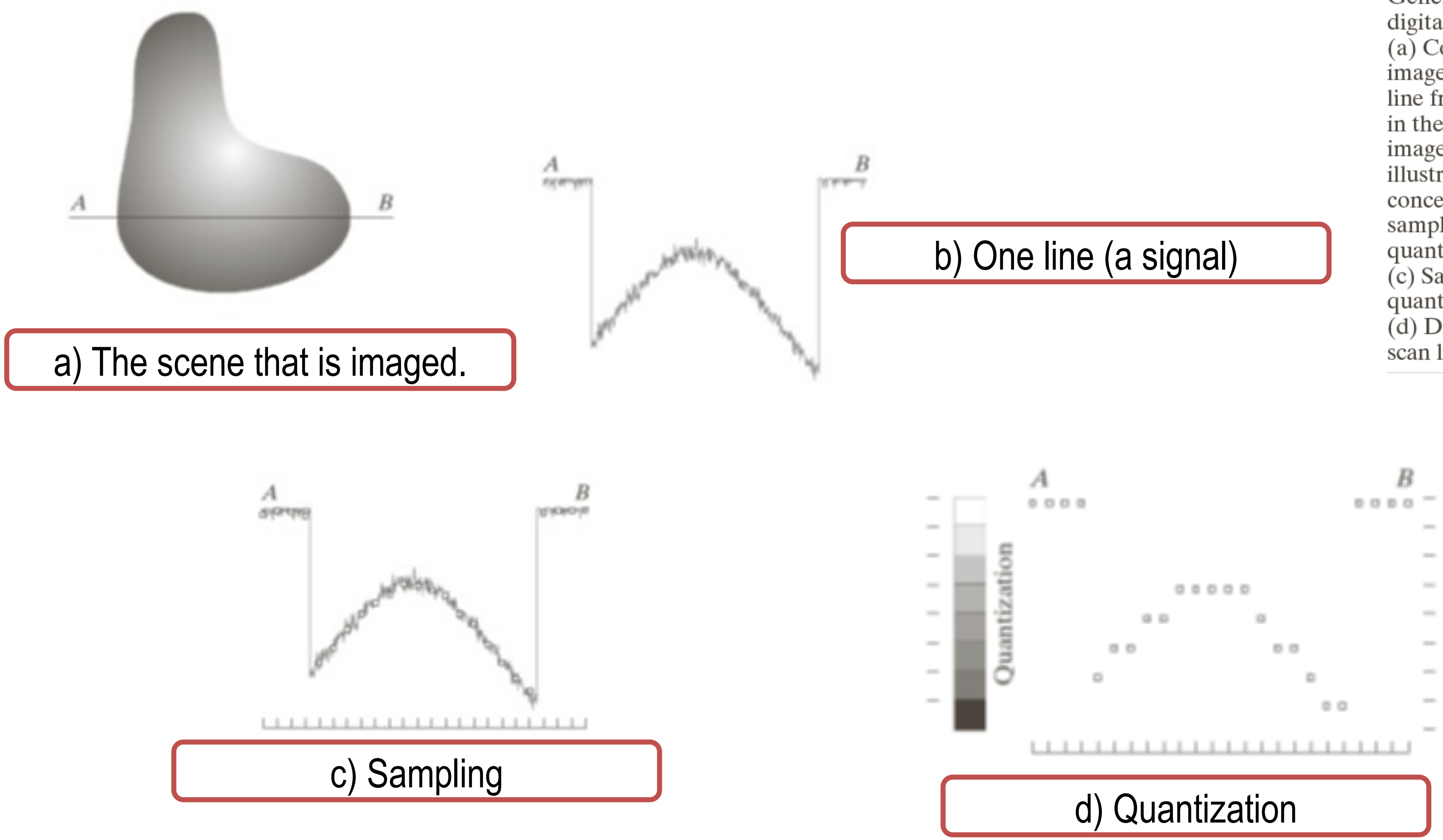
Three important coordinate systems:

1. *World* coordinates
2. *Camera* coordinates
3. *Image* coordinates

How do we project a given world point (x, y, z) to an image point?

Digitalization = Sampling + Quantization

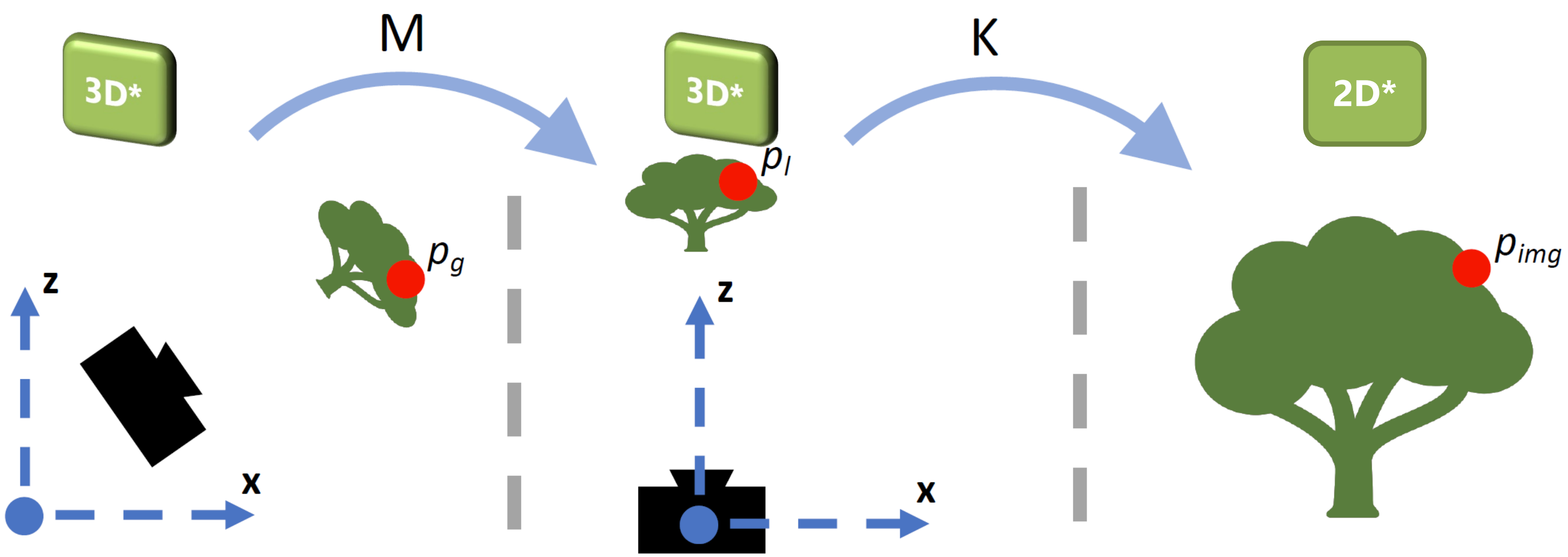
- Sampling: digitizing coordinate values
- Quantization: digitizing amplitude values



a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Coordinate frames



World coordinates

Camera coordinates

Image coordinates

Question: What do the indices g and l mean?

Question 2: Why the * at the dimensions?

Figure credit: Peter Hedman

Camera parameters

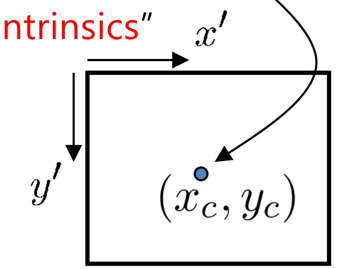
To project a point (x, y, z) in *world* coordinates into a camera

- First transform (x, y, z) into *camera* coordinates
- Need to know
  - Camera position (in world coordinates)
  - Camera orientation (in world coordinates)
- Then project into the image plane to get *image (pixel) coordinates*
  - Need to know camera *intrinsics*

Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principal point (c_x, c_y), pixel aspect size α
- blue parameters are called "extrinsics," red are "intrinsics"



Projection equation

$$\mathbf{x} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$

Note that these parameters „s" are not the same.

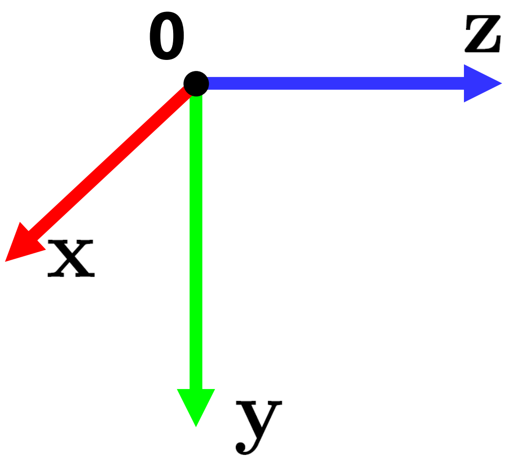
- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

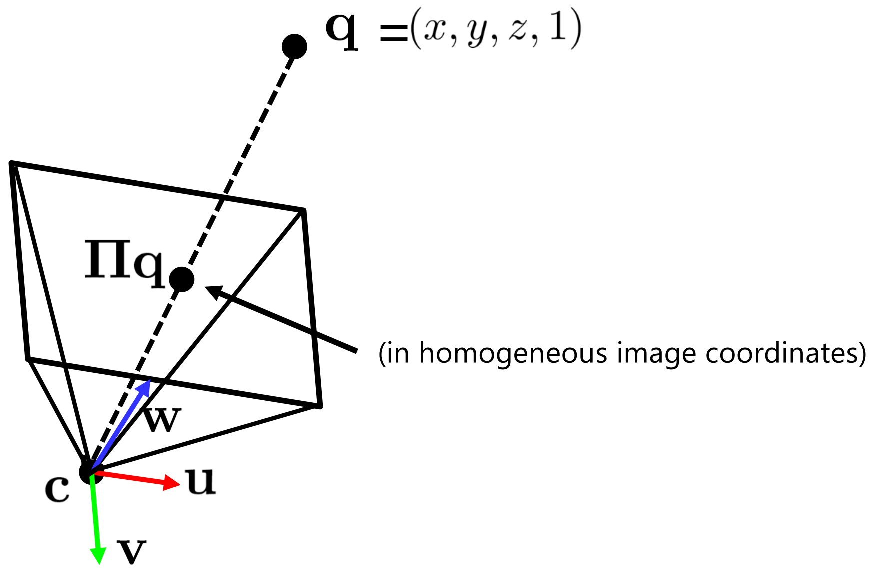
identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3\times3} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{T}_{3\times1} \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix}}_{\text{translation}}$$

- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

Projection matrix

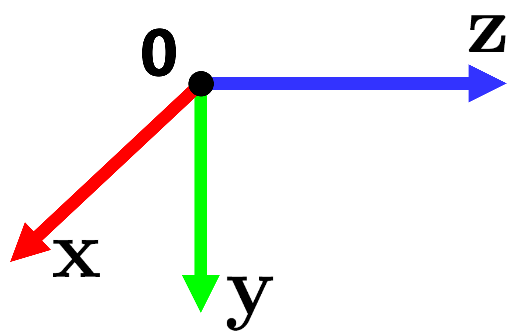




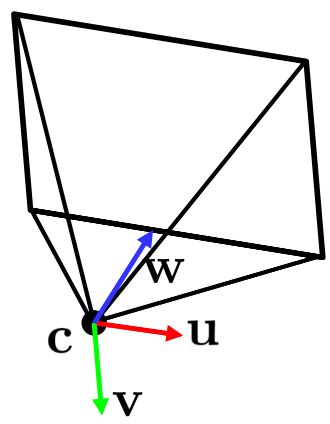
Extrinsics

- How do we get the camera to “canonical form”?

  = Center of projection at the origin, x-axis points right, y-axis points down, z-axis points forwards



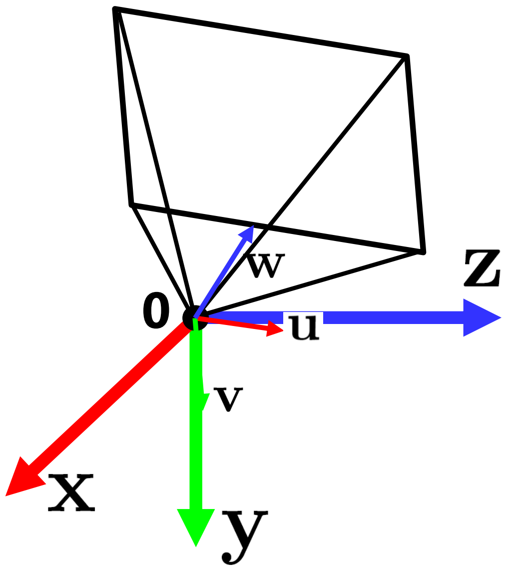
Step 1: Translate by -**c**



Extrinsics

- How do we get the camera to "canonical form"?

  = Center of projection at the origin, x-axis points right, y-axis points down, z-axis points forwards



Step 1: Translate by -**c**

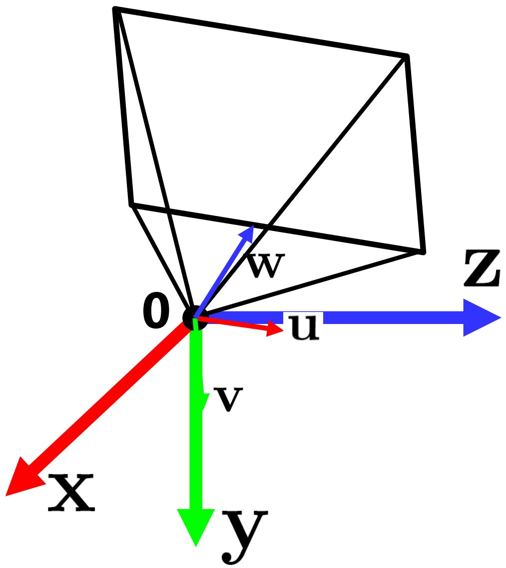
How do we represent translation as a matrix multiplication?

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

Extrinsics

- How do we get the camera to "canonical form"?

  = Center of projection at the origin, x-axis points right, y-axis points down, z-axis points forwards



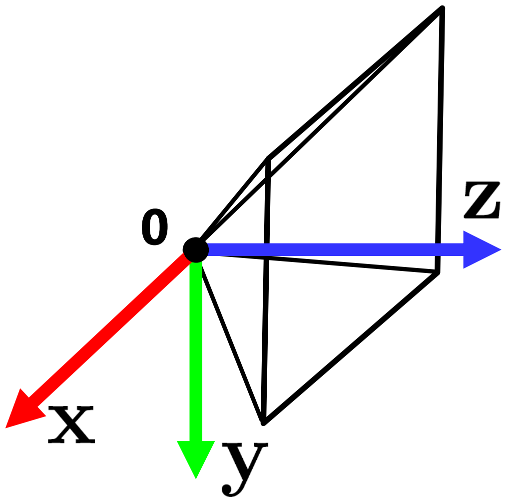
Step 1: Translate by -**c**
Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

3x3 rotation matrix

Extrinsics

- How do we get the camera to “canonical form”?
  = Center of projection at the origin, x-axis points right, y-axis points down, z-axis points forwards



Step 1: Translate by -**c**
Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

(with extra row/column of [0 0 0 1])

Perspective projection

$$\underbrace{\begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{\mathbf{K} \\ \text{(intrinsics)}}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

(c_x, c_y) : **principal point** ((w/2,h/2) unless optical axis doesn't intersect projection plane at image center)

Typical intrinsics matrix

$$\mathbf{K} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

- **2D affine transform** corresponding to a scale by f (focal length) and a translation by (c_x, c_y) (principal point)
- Maps 3D rays to 2D pixels

Focal length

- Can think of as “zoom”

24mm

50mm

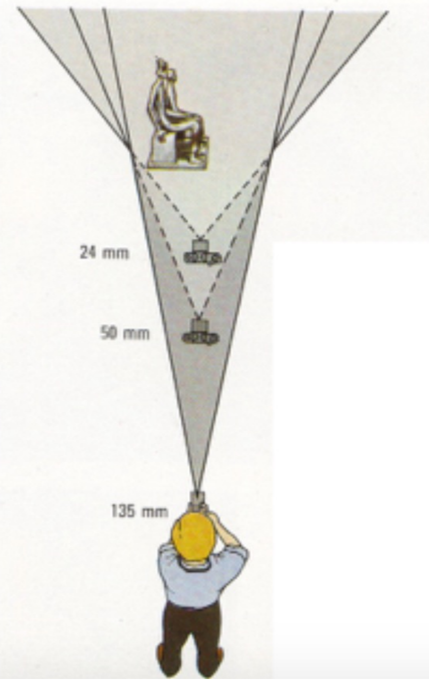
200mm

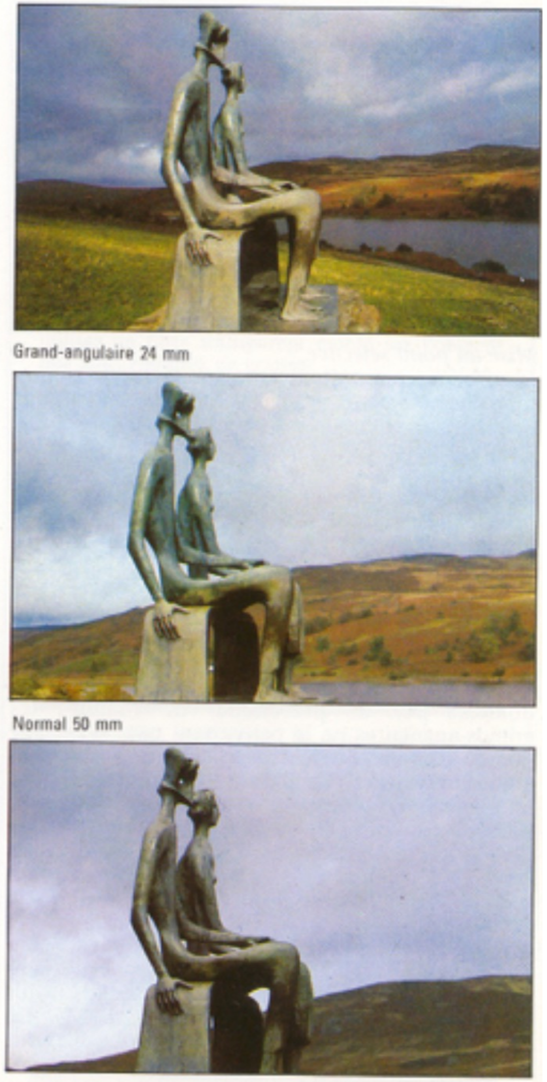
800mm

- Also related to *field of view*

Focal length vs. viewpoint

- Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.





Grand-angulaire 24 mm

Normal 50 mm

Longue focale 135 mm

Wide angle Standard Telephoto

Perspective distortion: Faces

- https://www.danvojtech.cz/blog/2016/07/amazing-how-focal-length-affect-shape-of-the-face/

http://petapixel.com/2013/01/11/how-focal-length-affects-your-subjects-apparent-weight-as-seen-with-a-cat/

Fredo Durand

Projection matrix

$$\boldsymbol{\Pi} = \underbrace{\mathbf{K}}_{\text{intrinsics}} \underbrace{\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} & & & 0 \\ & \mathbf{R} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} & \mathbf{I}_{3\times 3} & & -\mathbf{c} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{translation}}}$$

The **K** matrix converts 3D rays in the camera's coordinate system to 2D image points in image (pixel) coordinates.

This part converts 3D points in world coordinates to 3D rays in the camera's coordinate system. There are 6 parameters represented (3 for position/translation, 3 for rotation).

Projection matrix

$$\mathbf{\Pi} = \underbrace{\mathbf{K}}_{\text{intrinsics}} \underbrace{\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 \; 0 \; 0 & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ 0 \; 0 \; 0 & 1 \end{bmatrix}}_{\text{translation}}}_{\left[\, \mathbf{R} \,\middle|\, \underbrace{-\mathbf{R}\mathbf{c}}_{(\mathbf{t} \text{ in book's notation})} \,\right]}$$

$$\mathbf{\Pi} = \mathbf{K} \left[\, \mathbf{R} \,\middle|\, -\mathbf{R}\mathbf{c} \,\right]$$

Projection matrix